12.3 Calculating Probabilities

GOAL

Use tree diagrams and organized lists to calculate probabilities.

Learn about the Math

Carina, Kito, Rowyn, and Rishi are standing in line for the Drop Zone ride at Canada's Wonderland.

? If the ride operator loads them randomly, what is the probability that Rishi will sit between Rowyn and Carina?

A. Use a tree diagram to show all the possible seating arrangements.

B. Use your tree diagram to determine all the possible outcomes and all the **favourable outcomes**.

C. Calculate the probability that Rishi will sit between Rowyn and Carina.

D. What is the probability of the **complementary event** "Rishi will not sit between Rowyn and Carina"?

favourable outcome

a desired result when calculating a probability; for example, the result that Rishi will sit between Rowyn and Carina

complementary events

two events that have no outcome(s) in common but account for all possible outcomes of an experiment. The sum of the probabilities of complementary events is 1; for example, tossing Heads and tossing Tails are complementary events

Reflecting

1. How did you know that your tree diagram covered all the possible seating arrangements?

2. How did you know that the denominator of the probability in fraction form is 24?

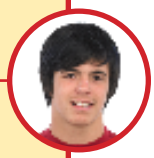
3. Explain how you used your tree diagram to calculate the probability that Rishi will sit between Rowyn and Carina.

4. Explain how you determined the probability in step D.

Work with the Math

Example 1: Using an organized list to calculate probability

Monica selected math, geography, science, and English for her first semester of high school. She will have the same teacher for geography and English, so she will not have to change classrooms if these subjects are back to back. What is the probability of geography and English being back to back?



Stefan's Solution

EMSG MSGE SEGM GEMS
EMGS MSEG SEMG GESM
EGMS MGSE SGEM GMES
EGSM MGES SGME GMSE
ESMG MESG SMEG GSEM
ESGM MEGS SMGE GSME

I used an organized list to make sure that I looked at all the possibilities.

There are four possible subjects for period 1, so each of the four columns starts with a different subject.

There are three possibilities for period 2, because one subject was already in period 1.

There are two possibilities left for period 3, because two subjects are already in periods 1 and 2.

There is only one possibility left for period 4, because three subjects are already in periods 1, 2, and 3.

From my list, I can see that 12 of the 24 possible schedules are favourable. The probability of English and geography being back to back is $\frac{12}{24}$, or $\frac{1}{2}$.

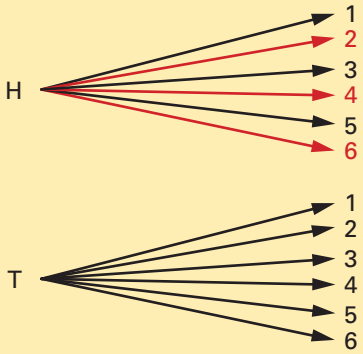
Example 2: Using a tree diagram to calculate probability

Suppose that you toss a coin and then roll a six-sided die.

a) What is the probability of getting Heads and an even number?

b) What is the probability of not getting Heads and an even number?

Kayley's Solution



I listed all the possibilities in a tree diagram.

The 12 branches represent 12 equally likely outcomes.

a) Three of the branches start with Heads and end in an even number, so three outcomes are favourable.

$P(\text{H and even}) = \frac{3}{12}$, or $\frac{1}{4}$

b) Not getting Heads and an even number and the event in part (a) are complementary events.

$P(\text{not H and even}) + P(\text{H and even}) = 1$

$P(\text{not H and even}) + \frac{1}{4} = 1$

$P(\text{not H and even}) = \frac{3}{4}$

A Checking

5. Use the tree diagram you made in step A on page 408 to calculate each theoretical probability.

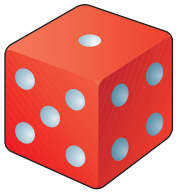
a) P(Rowyn beside Rishi)

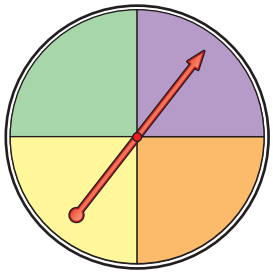
b) P(Rowyn not beside Rishi)

c) P(either Rowyn or Carina, but not both, on the outside)

B Practising

6. Suppose that you roll the die and spin the spinner.





Use an organized list to determine the probability of each event.

a) P(3 and yellow)

b) P(anything except 3 and yellow)

c) P(number $>$ 3 and purple)

7. a) Create a tree diagram to show all the possible outcomes for tossing three coins.

b) What is the probability of getting one Tail?

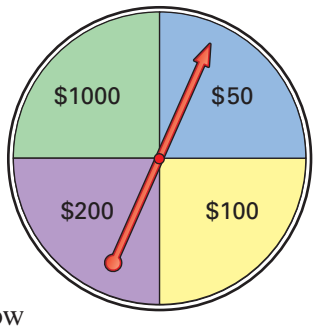
c) What is the probability of getting two or three Tails?

d) What is the probability of not getting any Heads?

8. How does a tree diagram give you the denominator of the fraction form of a probability? How does an organized list give you the denominator?

9. Calculate the probability of having three boys in a family with three children.

10. Kaycee has won a contest. To determine the amount of her prize, she must spin this spinner twice. She will receive the sum of her two spins.



a) Create a tree diagram to show all the possible outcomes.

b) What is the probability that Kaycee will receive more than the minimum amount but less than the maximum amount?

c) What is the probability that Kaycee will receive more than $500?

11. Anthony, Peter, Francis, and Christopher are in a race. The first three to finish will receive ribbons. Which is more probable—that both Anthony and Peter will receive ribbons, or that Peter will finish ahead of Francis and Christopher?

C Extending

12. Deanna and Carol are playing a game. They roll a die twice and add the numbers they roll. A sum of 5 scores a point.

a) What is the probability of rolling a sum of 5?

b) Deanna rolled a sum of 5 on her first turn. List the different ways that she could have done this.

c) What is the probability that, when Deanna rolled a sum of 5, the number on the first roll was greater than the number on the second roll?

13. To play a new board game, you roll a die. Every fourth square has a penalty if you land on it. What is the probability that you will get at least one penalty in your first two rolls of the die?